Spatial cointegration analysis in econometric modelling.

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Abstract.

A spatial variant of the times series based cointegration approach is introduced, covering themes as stationarity and non-stationarity of spatial series; spatial unit roots; spatial integration of order zero, one and higher; spatial cointegration; global equilibrium versus local disequilibria; and spurious regression. As a specific example, a 'surprising' negative cross-sectional correlation between unemployment ratios and out-commuting propensities reported in recent literature is suggested to be spurious, as the errors from a regression of the latter on the former are shown to be spatially non-stationary, thereby preventing meaningfull global equilibrium relationship between the two variables to be inferred.

1. Introduction.

During the last two decades applied economists attempting to estimate times series econometric models have been aware of difficulties that arise when unit roots are present in the data. To ignore this fact and proceed to estimate long-run models containing non-stationary variables may lead to serious problems, predominantly spurious results. While the use of differenced (and thus hopefully stationary) variables will avoid these spuriousities, important long-run information will be removed.

The purpose of the present paper is to illustrate that alike problems as well has relevance when estimation is based on spatial (i.e. cross-sectional) series. As large amounts of information is fastly and easily obtained at once from spatial sampling (opposed to times series sampling where one naturally have to wait for data to be born one piece after another for maybe several years) estimation based on spatial sampling has become widespread. As the estimation of a global model for an entire cross-section require global equilibrium, and thus spatial stationarity of the involved series, a proper investigation of the nonstationarity problem, its impact on the model, and the presence or absence of a global equilibrium relationship call for careful consideration.

Part 2 of the paper presents the notions of global versus local models, formalizes the concepts of spatial stationarity and nonstationarity, illustrates the nature of spurious regression, and introduces the terms spatial integration and spatial cointegration. Testing for spatial unit roots is considered in part 3, followed by suggestions for a spatial single-equation cointegration approach in part 4. Part 5 provides an illustrative example, and the paper is rounded off in part 6 with some concluding remarks and suggestions for future research.

2. Global and local models.

2.a. Global models.

Consider the following specific example to be examinated later: Based on a cross section of 275 Danish municipalities, the percentage of the workforce commuting to other municipalities (y) and the unemployment ratio (x) are observed for a specific year. Following labour market theory, y should be positively impacted by x. Apart from deciding which other impacting factors should be included, and omitting any discussion of (certainly relevant) nonlinear functional forms, the following linear regression is specified:

(1) $$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \text{ , } i = 1 \text{ , .. , } 275$$

where β_1 is supposed to have a fixed positive value. It is worth noticing that we have not made any assumption about whether spatial variation in x causes spatial variation in y. Equation (1) depicts an equilibrium situation in which all municipalities are able to react in precisely the same manner, i.e. fixing y at the level β_0 plus a fixed proportion of x, measured by $\beta_1 x$. If we were to assume that conditions outside the model (1) impacted the local propensity to commute via local adjustment processes, then local deviations from the equilibrium may emerge. Specifically, local rigidities in the process of full adjustment to x causes such local deviations from the global equilibrium. The further discussion of this item will be delayed to part 2.e, as some preliminaries have to be established in order to provide a proper formalization thereof.

2.b. Spatial stationarity and nonstationarity.

When considering estimation of a single equation like (1), it is important to consider the underlying properties of the processes that generate spatial variables. Specifically, it will be shown that models containing spatially non-stationary variables will often lead to a problem of spurious regression, whereby the results obtained suggest that there are statistically significant relationships between the variables in the regression model when in fact all that is obtained is evidence of spatial correlations rather than meaningful causal relations.

Starting with a very simple spatial data generating process, suppose that a variable is generated by the following first-order spatial autoregressive process:

$$(2) \qquad y_i = \rho y_L + \mu_i$$

where y_L is the average of y in regions spatially contingent to (i.e. neighbours to) region i. Thus, the actual value of the variable, y_i, depends on the average values of the variable in spatially contingent regions plus a disturbance term, μ_i, encapsulating all other random influences. It is assumed that this disturbance term follows a $N(0,\sigma^2)$ distribution and that the disturbances for any two regions are independent. For notational simplicity, (2) will be stated in vector-matrix notation: Let n be the number of regions. Define an n by n matrix **W** by $W_{ij} = 1$ iff region i and j are spatially contiguous, and 0 otherwise. The diagonal elements of **W** are set to 0. Finally, **W** is row-standardized, i.e. each element is divided by the number of 1's in the row to which the element belongs. Then (2) is equivalent to

$$(3) \qquad \begin{aligned} \mathbf{y} &= \rho \mathbf{W}\mathbf{y} + \boldsymbol{\mu} \\ \boldsymbol{\mu} &\sim N(\mathbf{0},\sigma^2\mathbf{I}) \end{aligned}$$

where **y** and **μ** are n-vectors and **I** the n by n identity matrix.

The variable **y** will be spatial stationary if the modulus of ρ is less than 1. If ρ approaches 1 then y will be spatial non-stationary. A spatial stationary series tends to fluctuate around its (global equilibrium) mean value within a more-or-less constant range (i.e. has a well defined variance) while a spatial non-stationary series varies around different means at different localities.

The question of whether a variable is spatial stationary depends on wether it has a unit root. Rewriting (3) as

$$(4) \qquad (\mathbf{I}-\rho\mathbf{W})\mathbf{y} = \boldsymbol{\mu}$$

it is clear that spatial stationarity requires the modulus of ρ to be less than 1. These results generalize easily to a higher order SAR(p) process; see Appendix 1 for details.

2.c. Spurious regressions.

Spatial trends in data can lead to spurious correlations implying relationships between the variables in a regression equation, when all that is present are correlated spatial trends. Consider the following simple spatial data generating process:

$$(5) \qquad \begin{aligned} \mathbf{y} &= \rho_y \mathbf{W}\mathbf{y} + \varepsilon, \quad \varepsilon \sim N(\mathbf{0},\mathbf{I}) \\ \mathbf{x} &= \rho_x \mathbf{W}\mathbf{x} + \nu, \quad \nu \sim N(\mathbf{0},\mathbf{I}) \end{aligned}$$

where ρ_y and ρ_x comes close to 1. In this formulation, **y** and **x** are uncorrelated and spatially non-stationary variables such that when the regression (1) is estimated, it should generally be accepted that β_1 is 0. However, because of the nonstationary nature of data, implying nonstationarity of ,, the tendency for both variables to vary according to the same spatial sceme leads to correlation which is picked up by the regression model.

To illustrate the problems of spurious correlation, a simulation study has been undertaken where (1) was estimated 1000 times with **x** and **y** as defined in (5) and L and $<$ generated by a standard normal distribution random number generator. The estimations as well as the data generating process used the 275 by 275 neighbour matrix for the Danish municipalities as **W**. For each of the 1000 replications, the t value for β_1 were calculated, and p(reject) reports the share of the 1000 replications in which the null of $\beta_1 = 0$ was (spuriously) rejected at the 5 percent level. The following results emerged for varying values of ρ_y and ρ_x.

ρ_y (= ρ_x)	p(reject)
0	0.056
0.2	0.052
0.4	0.063
0.6	0.119
0.8	0.221
0.9	0.320
0.99	0.435
0.999	0.441
0.9999999	0.492
1	0.490

For ρ_y and ρ_x close to zero, a desireable behaviour was found, with a reject probability close to 5 percent. For ρ_y and ρ_x rising - especially to more than 0.5 - the rejection probability rises drastically, until close to 50 percent for ρ_y and ρ_x approaching 1. Concluding, the problem of falsely inferring a causal relationship between two unrelated non-stationary series is quite serious. This leads to the question of when it is possible to infer causal global relationships between non-stationary spatial series, based on estimating a standard regression as (1).

2.d. Spatial cointegration.

If a series must be spatially differenced d times before it becomes spatially stationary then it contains d unit roots and is said to be spatial integrated of order d, denoted SI(d). Considering two series, **x** and **y**, which are both SI(d), any linear combination of these is in general SI(d). If, however, a spatial cointegrating vector β exists such that the residuals from the regression L = **y**-**x**β is of lower order of spatial cointegration, say SI(d-b) where $0 < b \leq d$, then **y** and **x** is spatial cointegrated of order (d,b) denoted SCI(d,b). If **y** and **x** where both SI(1) and ε SI(0) then **x** and **y** would be SCI(1,1).

The intuitive interpretation of spatial cointegration is that if two series are linked to form a global equilibrium then even though the series themselves may contain local trends (implying non-stationarity) they will nevertless move close together over space and the difference between them will be stable (stationary).
An illustrative example is easily provided. Let $\mathbf{x}=(\mathbf{I}-\mathbf{W})^{-1}<$, with < an n-vector of random N(0,1) numbers, and $\mathbf{y}=a+b\mathbf{x}+\mathbf{e}$, with **e** defined alike <. The coefficients a and b are arbitrarily chosen numbers (without loss of generality, let a=0 and b=1). Then the residual, **e**, from a regression of **y** on **x** will clearly be SI(0) whereas **y** and **x** per construction are SI(1) and also SCI(1,1). In this case, the estimated b expresses the global equilibrium relationship between **x** and **y**, which is also clear from the construction of these variables.

Thus, the identification of spatial cointegration gives sense to regressions involving spatially non-stationary variables. If these are spatially cointegrated then regression analysis imparts meaningful information about global relationships whereas if spatial cointegration is not established we return to the problem of spurious correlation.

2.e. Local models.

An equation like (1) sets out the global relationship governing, to keep the illustrative example in mind, the commuting propensity. However, even assuming that it is possible to estimate this global model directly it is also of interest to consider the local evolution of the variables under consideration, especially since global equilibrium may rarely be observed in any local neighbourhood. This is important especially because of the information that can be obtained from considering the dynamics of adjustment from local (dis)equilibrium to global equilibrium.

The major reason why relationships are not always in equilibrium centres - inter alia - on the inability of agents and phenomena to adjust to new information instantaneously. There are often substantial costs or other rigidity barriers of adjustment, causing the observed y_i to be determined not only by lagged values of y_i, i.e. $(\mathbf{Wy})_i$, $(\mathbf{W}^2\mathbf{y})_i$ etc., but also lagged values of x_i, i.e. $(\mathbf{Wx})_i$, $(\mathbf{W}^2\mathbf{x})_i$ etc. A simple spatial dynamic model with lags p=q=1 of local adjustment is:

(6) $$\mathbf{y} = \square_0 + \square_0\mathbf{x} + \square_1\mathbf{Wx} + \square_1\mathbf{Wy} + L$$

with L defined as a white noise. Rewriting (6) as

(7a) $$\mathbf{y} = \{\square_0/(1-\square_1)\} + \{(\square_1+\square_0)/(1-\square_1)\}\mathbf{x} + \{-\square_1/(1-\square_1)\}\,ª\mathbf{x} + \{-\square_1/(1-\square_1)\}\,ª\mathbf{y} + \{1/(1-\square_1)\}L$$

or

(7b) $$\mathbf{y} = \square_{0*} + \square_{*} + \square_1\,ª\mathbf{x} + \square_2\,ª\mathbf{y} + <$$

where ª$=(\mathbf{I}-\mathbf{W})$, clearly shows that the global elasticity ($\square_1$) between **y** and **x** is clearly different from the local elasticity ($\square_0$) as the former equalizes the term $\square_* = \{(\square_1+\square_0)/(1-\square_1)\}$, assuming $\square_1 < 1$ to ensure convergence of the local model to a global solution. The spatial model (6-7) is easily generalized to allow for more realistic adjustment processes by increasing p and q. However, this form of model may cause several problems. First, the likely high correlation between actual and lagged variables may cause multicollinearity (high R_2, imprecise parameter estimates, low t-values). A Hendry-type general-to-specific approach migth therefore result in misspecification (expecially if more than one explanatory variable is included). Second, the former mentioned problem of nonstationarity, common trends and spurious regression may increase. Presumeably, a solution migth be to respecify the spatial model, in terms of spatial first differences. But this removes any information about the global equilibrium and the adjustment processes to this. Alternatively, a spatial error-correction model may be helpful. The formulation (6-7) is equivalent to

(8) ª$\mathbf{y} = \square_0$ ª$\mathbf{x} - (1-\square_1)(\mathbf{Wy} - \square_{0*} - \square_{1*}\mathbf{Wx}) + L$

where $\square_{0*}=\square_0/(1-\square_1)$, and $\square_{1*}=-(\square_0+\square_1)/(1-\square_1)$, which incorporates both local and global effects. The term $(\mathbf{Wy} - \square_{0*} - \square_{1*}\mathbf{Wx})$ will equal 0 in case of global equilibrium, and, in case of local disequilibrium, measure the distance from local disequilibrium to global equilibrium. An estimate of $(1-\square_1)$ provides information on the speed of adjustment to global equilibrium.

A second feature of the spatial error-correction model (8) is that all terms are stationary, provided cointegration and estimates of $\square_{0*}$ and $\square_{1*}$.

Third, the spatial error-correction model is closely bound up with the concept of spatial cointegration, as **x** and **y** being SCI(1,1) is equivalent to the existence of a spatial error-correction model, thereby providing this model with immunity from the spurious regression problem.

Finally, the error-correction model is easily generalized to general lag lengths p and q, reformulating (8) as

(9) $\mathbf{A}(\mathbf{W})$ ª$\mathbf{y} = \mathbf{B}(\mathbf{W})$ ª$\mathbf{x} - (1-\square)(\mathbf{W}_p\mathbf{y} - \square_{0*} - \square_{1*}\mathbf{W}_p\mathbf{x}) + L$

where $\mathbf{A}(\mathbf{W})$ and $\mathbf{B}(\mathbf{W})$ are spatial lag polynomials on the form

$$\mathbf{A}(\mathbf{W}) = \mathbf{I} - \square_1\mathbf{W} - \square_2\mathbf{W}_p - .. - \square_p\mathbf{W}_p,$$
$$\mathbf{B}(\mathbf{W}) = \mathbf{I} - \square_1\mathbf{W} - \square_1\mathbf{W}_2 - .. - \square_q\mathbf{W}_q,$$

and $\square = \square_1 + \square_2 + .. + \square_p$.

3. Testing for unit roots.

The present investigation will focus on a spatial variant of the Dickey and Fuller (1979) approach to testing the null hypothesis that a spatial series contains a unit root against the alternative of stationarity. The simplest form of a spatial Dickey-Fuller test amounts to estimating

$$(10a) \qquad \mathbf{y} = \rho\mathbf{Wy} + \varepsilon$$

or, equivalently,

$$(10b) \qquad \Delta\mathbf{y} = (\mathbf{I}-\mathbf{W})\mathbf{y} = (\rho-1)\mathbf{Wy} + \varepsilon = \rho_*\mathbf{Wy} + \varepsilon$$

where ε is a white noise. The first specification is for H0 : $\rho=1$ against H1 : $\rho<1$. The latter is for H0 : $(\rho-1)=\rho_*=0$ against H1 : $\rho_*<0$, thereby simplifying matters when more complicated SAR(p) specifications replaces the simple SAR(1) specification in (10a-10b). The standard approach to testing the hypothesis (10b) is to construct a t-test, which, under the null of nonstationarity, does not follow the standard t-distribution, but, rather, what we may denote a spatial Dickey-Fuller distribution. This distribution and the critical values thereof must be computed using Monte Carlo techniques. A complication, opposed to the times series variant, is that the distribution will be different for different choices of contiguity matrix, **W**, so that the distribution must be derived for each specific application. However, what is more serious opposed to the times series variant, is that the OLS estimate of the t value will generally be not only inefficient but also inconsistent due to the correlation between the regressor and the error term in (10b), incorporated per construction of **Wy**. Consequently, ρ_* must be estimated using consistent methods. The present investigation applies a consistent and efficient maximum likelihood estimation, leading to a consistent and efficient estimate of t, which follows an asymptotic t (i.e. standard normal) distribution under the null of nonstationarity. Comparing this t value to the lower α percentile of the N(0,1) leads to accept (rejection) of the null at level α if t is higher (lower) than the chosen α percentile. For details on the maximum likelihood estimation, see the derivation in Appendix 2.

If a simple SAR(1) model is used when in fact **y** follows an SAR(p) process, then the error term will be spatially autocorrelated to compensate for the misspecification of the spatial structure of **y**. In the case of y being SAR(p),

$$(11a) \qquad \mathbf{y} = \rho_1\mathbf{Wy} + \rho_2\mathbf{W}_2\mathbf{y} + .. + \rho_p\mathbf{W}_p + \varepsilon$$

or

$$(11b) \qquad \Delta\mathbf{y} = \rho_*\mathbf{Wy} + \rho_{1*}\Delta(\mathbf{Wy}) + \rho_{2*}\Delta(\mathbf{W}_2\mathbf{y}) + .. + \rho_{p-1*}\Delta(\mathbf{W}_{p-1}\mathbf{y}) + \varepsilon$$

where $\rho_* = (\rho_1+\rho_2+..+\rho_p-1)$. If $\rho_* = 0$ against $\rho_* < 0$ holds then **y** contains a unit root. The above procedure might be applied to (11b) and the calculated t-value for ρ_* compared to the critical value as above.

4. Spatial cointegration in single equations.

If two series **x** and **y** are spatially cointegrated SCI(1,1) then estimating the global relationship between **x** and **y** amounts to estimate the model

(12) $\mathbf{y} = □_0 + □\mathbf{x} + ,$

because the error term will then be stationary SI(0). Recalling the local model (6) rewritten as (7a-7b) it is evident that estimating the global model is equivalent to estimating the local model without local terms î**x** and î**y** because the SI(1) variables **y** and **x** globally dominates the SI(0) variables î**x**, î**y** and ,. For the moment, we ignore the problem of , eventually being spatially autocorrelated by the omission of î**x** and î**y**. To test the hypothesis that **y** and **x** are not cointegrated amounts to test whether , is SI(1) against , being SI(0). For this purpose, the spatial Dickey-Fuller test suggested earlier will be applied.

Though this has to suffice for the time being, several methodological problems call for attention. First, if the parametres in (12) is not known in advance but obtained from estimating (12) then, due to the minimization feature, the estimated errors will appear as more stationary than they really are, tending the SDF test to overreject the null of nonstationarity as □* will be systematically downward biased. This may be partly resolved by incorporating a variant of the Engle and Yoo (1991) three step procedure. Second, the potential number of regressors included in (12) may in any empirical relevant case vary, whereby the critical value of the SDF test should vary with this number. Leaving a further discussion of this problem to future research, a possible solution might be a spatial variant of the MacKinnon (1991) response surfaces for critical values of the SDF t values. Another solution might be to formulate the log likelihood and concentrate □ away, whereby □* can be estimated consistently by a maximum likelihood procedure. Third, the common factor problem stated by Kremers, Ericsson and Dolado (1992) also applies here. The common factor problem emerges because the SDF test implies a restricted spatial ECM which can only equal the unrestricted ECM if the local and global reactions of **y** on changes in **x** are equal, which will only occur if the model is in equilibrium. As pointed out by the authors, if this restriction is invalid (as is often likely) a loss of information (and so of power) is imposed.

Based on the estimated global relationship between **y** and **x**, and establishing that , is SI(0), information on the speed of adjustment from local disequilibrium to global equilibrium may be derived using the spatial error-correction model approach. That is, having obtained **e** as an estimate for , from (12) it is possible to estimate

(13) î$\mathbf{y} = □_0$ î$\mathbf{x}$ - (1-□)$\mathbf{We}$ + L

where (1-□) measures the speed of adjustment to global equilibrium. Parallel to the suggestions in Kremers, Ericsson and Dolado (1992) and Banerjee et.al. (1993), a value of

□ close to zero indicates stationarity, whereas a value close to one indicates absence of a global equilibrium (i.e. no spatial cointegration.

A spatial variant of the Engle and Yoo (1991) three step approach is provided by the following steps:

1. Estimate the global model (12), obtain estimated errors, denored **e**, and a first stage estimate of β, denoted β_1.

2. Estimate the ECM (13), using **e**, and obtain estimate of (1-□), denoted (1-a), and estimated errors, denoted **u**.

3. Regress **u** on the spatially lagged explanatory variables, multiplied by (1-a), that is

$$\mathbf{u} = [(1-a)\mathbf{Wx}]\delta + \varepsilon .$$

The estimated δ and its standard deviation (being the standard deviation for β_3 below) is obtained. Calculate the corrected estimator as $\beta_3 = \beta_1 + \delta$.

Using this estimator (and its standard deviations for the calculation of t values) in the global model (12) provide residuals which may be tested for a unit root. Essentially, this procedure is expected to lower the bias in the estimate of β, whereby the inconsistency of the SDF approach is lowered.

Before closing this section, a final serious problem calls for attention. If the global model contains several variables, there may be more than one cointegration vector. Specifically, with k variables in the model up to k-1 linearly independent cointegration vectors may exist. Assuming that there is only one cointegration vector when in fact there are more, leads to inefficiency in the sense that we can only obtain a linear combination of these vectors when estimating a single equation model. This inefficiency is further extended if the causality in the model is more complicated than suggested by the single equation formulation. Specifically, for the single equation approach to be valid, one must assume that only one cointegration vector is present and that all right-hand-side variables are exogenous. The strong duality of these problems calls for a spatial adoption of a multivariate procedure (including a methodology to test for exogeneity) alike the VAR approach developed for the temporal case by inter alia Johansen (1988).

5. Empirical illustrations.

To provide a brief illustration of the spatial cointegration approach suggested above, a slighly simplified version of a model for commuting, suggested by Lauridsen and Nahrstedt (1998), will be discussed. In short, this model states

$$\text{OUTCOM} = \beta_0 + \beta_1 \text{*PSH1766} + \beta_2 \text{*WORKPL} + \beta_3 \text{*IPHOUS} + \beta_4 \text{*UNEMP}$$

where

OUTCOM: Number of persons with residence in the municipality and workplace in another municipality in percentage of the number of workplaces in the municipality, 1994 (Source: Danish Statistical Bureau);
WORKPL: Number of workplaces per 100 inhabitants, 1994 (Source: Danish Statistical Bureau);
PSH1766: Population share of 17-66 year-olds, 1994 (Source: Danish Statistical Bureau);
IPHOUS: Inhabitants per household, 1994 (Source: Danish Statistical Bureau); and
UNEMP: Unemployed per 100 17-66 year-olds, 1994 (Source: Danish Statistical Bureau);

all variables observed for the 275 Danish municipalities in 1994. A priori, β_1, β_3 and β_4 are expected to be positive, whereas β_2 is expected to be negative. However, estimating the equation using OLS gave the following result:

OUTCOM = -264.6 + 6.29*PSH1766 - 2.33*WORKPL + 18.78*IPHOUS - 3.39*UNEMP
(7.92) (17.58) (23.11) (2.34) (6.49)
$R^2 = 0.81$

where the t values in parentheses as well as the R^2 value indicate a good model fit. What is more disturbing is that the sign of the estimate for the UNEMP coefficient contradicts the a priori assumption. Following the discussion hitherho, this contradiction may be resolved by different arguments.

First, one may argue that the a priori assumption about the sign of β_4 is wrong. However, as this is not in accordance with the general belief it is not a very convincing argument, so we will refrain from discussing it further. Second, one may establish evidence that the estimated relationship is spurious i.e. that, due to rigidities in the local adjustment of OUTCOM to UNEMP the global equation has a non-stationary error process thereby precluding the existence of a global equilibrium relationship between these variables. Formally, the integration orders of the employed variables and the estimated errors should be investigated. However, before passing on to this, a third problem should be considered. One may argue that exogeneity of UNEMP is unrealistic. It may as well be that variation in OUTCOM causes variation in UNEMP (which is consistent with the negative correlation between these two variables). This calls for a consideration of a spatial multivariate (multiequation) approach as well as a spatial consideration of the exogeneity problem. Due to the present lack of methodology, however, these questions will have to await future discussion.

An application of the simple and the consistent spatial Dickey-Fuller approach to each variable in the model (including the error term) gave the following results:

Variable	*		*	
	(Simple)		(Consistent)	

OUTCOM	-0.40	-4.77***			-0.61	-1.73**	
UNEMP	-0.03	-0.69		-0.26	-1.08		
PSH1766	-0.07	-1.48*		-0.28	-1.05		
WORKPL	-0.79	-7.69***			-0.88	-2.85***	
IPHOUS		-0.32	-4.03***			-0.55	-1.38*
error	-0.32	-3.90***			-0.56	-1.47*	

where 1, 2 and 3 asterisks marks significanse at the 10, 5 and 1 percent level according to the asymptotic t distribution. The discrepancies between the simple and the consistent procedure is obvious, as the former tends to overreject the null of spatial nonstationarity. The results from applying the latter is that UNEMP, PSH1766 and IPHOUS are spatially nonstationary. Especially, as OUTCOM is stationary, one would not expect the error to be stationary. This is confirmed, as the null of nonstationarity for the error term cannot be rejected at the 5 percent level.

Estimating the local model (13) gave the following results:

ªOUTCOM = -3.43* ªUNEMP + 4.15* ªPSH1766 - 2.16* ªWORKPL + 59.55* ªIPHOUS - 0.25***We**
(-5.64) (9.19) (-26.45) (7.83) (-3.40)
R2 = 0.87

where the estimated value of [illegible] is close to 0.75. Though it is not clear how close this is to 1, the fairly high value provides further evidence of the spatial nonstationarity and thus the absense of a global equilibrium model.

Usage of the spatial Engle-Yoo procedure provided the following updated global model:

OUTCOM = -264.63 - 4.08*UNEMP +7.96*PSH1766 - 1.90*WORKPL -31.89*IPHOUS
[-0.70] [1.67] [0.43] [-50.67]
(-7.92) (-2.36)(7.28) (-4.09) (-1.28)
R2 = 0.71

where the estimated values of * appear in square brachets. The [illegible] test for a unit root in the error term was here calculated as -3.24, which, opposed to the value for the uncorrected global model, rejects the nonstationarity hypothesis. However, the updated model does not solve the problem of the erratic sign of the UNEMP variable, but rather strengthen this problem by imposing a wrong sign on the IPHOUS variable, emerging from a very large adjustment of this coefficient.

6. Discussion and future research problems.

This investigation provides the first steps toward a spatial cointegration approach in spatial econometric modelling. A spatial variant of the Dickey-Fuller test for unit roots is suggested, followed by a spatial adoption of the Engle-Granger cointegration approach. However several problems is left open and call for future research. First, a consistent approach for estimating and evaluating the spatial Dickey Fuller test for a unit root in the error term of a regression model must be developed, including a resolution of the problem of correcting for the number of explanatory variables. Second, the higher order SAR(p) process with potentially more than one unit root merits closer attention. Third, the possible presence of spatially autocorrelated errors, causing inconsistent SDF results, must be resolved. Finally, a diversity of problems pertaining to adoption of the Engle-Granger approach into a spatial cointegration methodology need further investigation. Outstanding among these are the exogeneity problem, which must be given a spatial variant, and the problems related to the potential presence of more than one cointegration vector, suggesting a spatial adoption of the Johansen approach.

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Appendix 1. Unit roots and spatial stationarity.

Consider the general pth order SAR(p) process:

$$(1)\qquad \begin{aligned}\mathbf{y} &= \phi_1\mathbf{Wy} + \phi_2\mathbf{W}(\mathbf{Wy}) + \phi_3\mathbf{W}(\mathbf{W}(\mathbf{Wy})) + \dots + \phi_p\mathbf{W}(\mathbf{W}(\dots(\mathbf{Wy})\dots)) + \boldsymbol{\mu}\\ &= \phi_1\mathbf{Wy} + \phi_2\mathbf{W}_2\mathbf{y} + \phi_3\mathbf{W}_3\mathbf{y} + \dots + \phi_p\mathbf{W}_p\mathbf{y} + \boldsymbol{\mu}\end{aligned}$$

which may be notationally simplified as

$$(2)\qquad Q(\mathbf{W})\mathbf{y} = \boldsymbol{\mu}$$

where $Q(\mathbf{W}) = \mathbf{I} - \square_1\mathbf{W} - \square_2\mathbf{W}_2 - \square_3\mathbf{W}_3 - .... - \square_p\mathbf{W}_p$.

Forming the characteristic equation for (1)-(2), $|Q(\mathbf{W})| = 0$, if all roots have moduli greater than 1 then **y** is spatial stationary. Thus, in case of a SAR(3) process,

(3) $$(\mathbf{I} - \square_1\mathbf{W} - \square_2\mathbf{W}_2 - \square_3\mathbf{W}_3 - \square_3\mathbf{W}_3)\mathbf{y} = \boldsymbol{\mu}$$

and if a unit root exists then (3) can be factorized into $(\mathbf{I} + \square\mathbf{W} + \square\mathbf{W}_2)(\mathbf{I}\text{-}\mathbf{W})\mathbf{y} = \boldsymbol{\mu}$, where $\square$ and $\square$ depends on the $\square_i$'s. If there is only one unit root then the roots of $|\mathbf{I} + \square\mathbf{W} + \square\mathbf{W}_2|$ must have moduli greater than unity. Then $\hat{\imath}\mathbf{y} = (\mathbf{I}\text{-}\mathbf{W})\mathbf{y}$ must be a stationary process. If there are two unit roots then we can further factorize (3) into $(\mathbf{I}+\square\mathbf{W})(\mathbf{I}\text{-}\mathbf{W})(\mathbf{I}\text{-}\mathbf{W})\mathbf{y} = \boldsymbol{\mu}$, where $\square$ depends on $\square$ and $\square$. If $\square$ has modulus greater than unity, the second difference $\hat{\imath}_2\mathbf{y} = (\mathbf{I}\text{-}\mathbf{W})(\mathbf{I}\text{-}\mathbf{W})\mathbf{y}$ is stationary. For the general SAR(p) process, if m unit roots are present, then $\hat{\imath}_m\mathbf{y} = (\mathbf{I}\text{-}\mathbf{W})_m\mathbf{y}$ is stationary.

Consider the following reformulation of the SAR(p) process:

(4) $$(\mathbf{I}\text{-}\mathbf{W})\mathbf{y} = \square_*\mathbf{W}\mathbf{y} + \square_{1*}(\mathbf{I}\text{-}\mathbf{W})\mathbf{W}\mathbf{y} + \square_{2*}(\mathbf{I}\text{-}\mathbf{W})\mathbf{W}_2\mathbf{y} + .. + \square_{p*}(\mathbf{I}\text{-}\mathbf{W})\mathbf{W}_p\,\mathbf{y} + \boldsymbol{\mu}$$

where $\square_* = \square_1 + \square_2 + .. + \square_p - 1$. In the SAR(3) case with at least one unit root (3) reduces to

(5) $$(\mathbf{I}\text{-}\mathbf{W})\mathbf{y} = -\square(\mathbf{I}\text{-}\mathbf{W})\mathbf{W}\mathbf{y} - \square(\mathbf{I}\text{-}\mathbf{W})\mathbf{W}_2\mathbf{y} + \boldsymbol{\mu}\ .$$

Comparing the SAR(3) version of (4) with (5) shows that $\square_* = 0$ if there is a unit root such that $\square_1 + \square_2 + \square_3 = 1$. If $\square_* < 0$, then $\square_1 + \square_2 + \square_3 < 1$ and **y** must be spatially stationary. These arguments are easily generalized to the SAR(p) case, such that a test for stationarity need only consider the hypothesis $\square_* = 0$ against the alternative $\square_* < 0$.

Appendix 2. A consistent spatial Dickey Fuller test.

The model to be estimated reads as

$$(\mathbf{I}\text{-}\mathbf{W})\mathbf{y} = \square_*\mathbf{W}\mathbf{y} + \mathbf{u}\ ,\ \mathbf{u} \ \square\ \mathrm{N}(\mathbf{0},\square_2\mathbf{I})$$

or

$$(\mathbf{I}\text{-}\mathbf{W}\text{-}\square_*\mathbf{W})\mathbf{y} = \mathbf{A}\mathbf{y} = \mathbf{h}(\mathbf{y}) = \mathbf{u}.$$

With the likelihood for **u** specified as

$$\mathrm{L}_u(\mathbf{u}) = (2\square)_{-n/2}(\square_2\mathbf{I})_{-n/2}\exp(-1/(2\square_2)\mathbf{u}'\mathbf{u})$$

the likelihood for **y** becomes

$$L_y = L_u(\mathbf{h}(\mathbf{y}))|\partial \mathbf{h}(\mathbf{y})/\partial \mathbf{y}|$$
$$= (2\pi)^{-n/2}(\sigma^2\mathbf{I})^{-n/2}\exp(-1/(2\sigma^2)\mathbf{y}'\mathbf{A}'\mathbf{A}\mathbf{y})|\mathbf{A}|$$

such that the log likelihood to be maximized with respect to ρ^* and σ^2 is

(1) $L = \log(L_y) = -(n/2)\log(2\pi) - (n/2)\log(\sigma^2) - 1/(2\sigma^2)\mathbf{y}'\mathbf{A}'\mathbf{A}\mathbf{y} + \log(|\mathbf{A}|)$

giving the first order conditions

(2) $\partial L/\partial(\sigma^2) = (-n/2)(\sigma^2)^{-1} + (1/2)\mathbf{y}'\mathbf{A}'\mathbf{A}\mathbf{y}(\sigma^2)^{-2} = 0$

and

(3) $\partial L/\partial \rho^* = -1/(2\sigma^2)\mathbf{y}'[\partial(\mathbf{A}'\mathbf{A})/\partial \rho^*]\mathbf{y} + \partial \log(|\mathbf{A}|)/\partial \rho^*$
$= -1/(2\sigma^2)\mathbf{y}'[-\mathbf{W}'\mathbf{A}-\mathbf{A}'\mathbf{W}]\mathbf{y} + \mathrm{tr}(\mathbf{A}^{-1}\partial \mathbf{A}/\partial \rho^*)$
$= 1/(2\sigma^2)y'[\mathbf{W}'\mathbf{A}+\mathbf{A}'\mathbf{W}]\mathbf{y} - \mathrm{tr}(\mathbf{A}^{-1}\mathbf{W})$.

From (2) it follows that

(4) $\sigma^2 = (1/n)\mathbf{y}'\mathbf{A}'\mathbf{A}\mathbf{y}$,

which expresses σ^2 conditioned on ρ^*, whereas (3) cannot be solved analytically for ρ^*. Insertion of (4) in (1) gives the concentrated log likelihood as a function of one parameter:

$$L = (-n/2)\log(2\pi) - (n/2)\log((1/n)\mathbf{y}'\mathbf{A}'\mathbf{A}\mathbf{y}) - (1/2)((1/n)\mathbf{y}'\mathbf{A}'\mathbf{A}\mathbf{y})^{-1}(\mathbf{y}'\mathbf{A}'\mathbf{A}\mathbf{y}) + \log(|\mathbf{A}|)$$
$$= (-n/2)\log(2\pi) - (n/2)\log((1/n)\mathbf{y}'\mathbf{A}'\mathbf{A}\mathbf{y}) - (n/2) + \log(|\mathbf{A}|)$$
$$= \text{constant} - (n/2)\log(\mathbf{y}'\mathbf{A}'\mathbf{A}\mathbf{y}) + \log(|\mathbf{A}|)$$

such that the first order condition on ρ^* is equivalent to

$$\max_{\rho^*}(L) = \min_{\rho^*}[(n/2)\log(\mathbf{y}'\mathbf{A}'\mathbf{A}\mathbf{y}) - \log(|\mathbf{A}|)]$$

which may be solved by a linear search in the relevant interval from -2 to 0 (or any other iterative search method).

From (2) - (3) the second order conditions become

(5) $\partial^2 L/\partial(\sigma^2)^2 = (n/2)(\sigma^2)^{-2} - \mathbf{y}'\mathbf{A}'\mathbf{A}\mathbf{y}\,(\sigma^2)^{-3}$,

(6) $\partial^2 L/\partial(\rho^*)^2 = 1/(2\sigma^2)\mathbf{y}'(\mathbf{W}'[\partial \mathbf{A}/\partial \rho^*]+[\partial \mathbf{A}'/\partial \rho^*]\mathbf{W})\mathbf{y} - \partial \mathrm{tr}(\mathbf{A}^{-1}\mathbf{W})/\partial \rho^*$
$= 1/(2\sigma^2)\mathbf{y}'(\mathbf{W}'(-\mathbf{W}) + (-\mathbf{W}')\mathbf{W})\mathbf{y} - \mathrm{tr}([\partial \mathbf{A}^{-1}/\partial \rho^*]\mathbf{W})$
$=-(1/\sigma^2)\mathbf{y}'\mathbf{W}'\mathbf{W}\mathbf{y}) - \mathrm{tr}(\mathbf{A}^{-1}\mathbf{W}\mathbf{A}^{-1}\mathbf{W})$,

where we have used $\partial \mathbf{A}^{-1}/\partial \rho^* = -\mathbf{A}^{-1}(\partial \mathbf{A}/\partial \rho^*)\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{W}\mathbf{A}^{-1}$, and

(7) $$\partial^2 L/\partial(\sigma^2)\partial\rho^* = \partial[\partial L/\partial(\rho^*)]/\partial(\sigma^2) = (-1/2)(\sigma^2)^{-2}\mathbf{y}'(\mathbf{W}'\mathbf{A}+\mathbf{A}'\mathbf{W})\mathbf{y} .$$

With $\theta' = (\rho^*, \sigma^2)$, the covariance matrix becomes $\mathbf{V} = \mathrm{cov}(\theta) = [\mathbf{I}(\theta)]^{-1}$, where $\mathbf{I}(\theta) = -E[\partial^2 L/\partial\theta\partial\theta']$. The elements of the Fisher information matrix $\mathbf{I}(\theta)$ are derived as follows:

$$\begin{aligned}\mathbf{I}_{11} &= -E[-(1/\sigma^2)\mathbf{y}'\mathbf{W}'\mathbf{W}\mathbf{y}) - \mathrm{tr}(\mathbf{A}^{-1}\mathbf{W}\mathbf{A}^{-1}\mathbf{W})] \\ &= \mathrm{tr}(\mathbf{W}'\mathbf{W}\mathbf{A}^{-1}(\mathbf{A}^{-1})') - \mathrm{tr}(\mathbf{A}^{-1}\mathbf{W}\mathbf{A}^{-1}\mathbf{W}) ,\end{aligned}$$

as $E((1/\sigma^2)\mathbf{y}'\mathbf{W}'\mathbf{W}\mathbf{y}) = (1/\sigma^2)E(\mathrm{tr}(\mathbf{W}\mathbf{y})'\mathbf{W}\mathbf{y}) = (1/\sigma^2)E(\mathrm{tr}(\mathbf{W}'\mathbf{W})\mathbf{y}'\mathbf{y}) = (1/\sigma^2)\mathrm{tr}(\mathbf{W}'\mathbf{W}E(\mathbf{y}\mathbf{y}')) = (1/\sigma^2)\mathrm{tr}(\mathbf{W}'\mathbf{W}\sigma^2\mathbf{A}^{-1}(\mathbf{A}^{-1})')$, using that $\mathbf{y}=\mathbf{A}^{-1}\mathbf{u}$ gives $\mathbf{y}\mathbf{y}'=\mathbf{A}^{-1}\mathbf{u}\mathbf{u}'(\mathbf{A}^{-1})'$,
such that $E(\mathbf{y}\mathbf{y}')=\mathbf{A}^{-1}(\sigma^2\mathbf{I})(\mathbf{A}^{-1})'$. Using alike derivations,

$$\mathbf{I}_{22} = -E[(n/2)(\sigma^2)^{-2} - \mathbf{y}'\mathbf{A}'\mathbf{A}\mathbf{y}\,(\sigma^2)^{-3}] = (-n/2)(\sigma^2)^{-2} + n\,(\sigma^2)^{-2} = (n/2)(\sigma^2)^{-2}$$

emerges. Finally,

$$\begin{aligned}\mathbf{I}_{12} = \mathbf{I}_{21} &= -E[(-1/2)(\sigma^2)^{-2}y'(\mathbf{W}'\mathbf{A}+\mathbf{A}'\mathbf{W})\mathbf{y}] = (1/2)(\sigma^2)^{-2}E[\mathbf{y}'(\mathbf{W}'\mathbf{A}+\mathbf{A}'\mathbf{W})\mathbf{y}] \\ &= (1/2)(\sigma^2)^{-2}2\sigma^2\mathrm{tr}(\mathbf{A}^{-1}\mathbf{W}) = (\sigma^2)^{-1}\mathrm{tr}(\mathbf{A}^{-1}\mathbf{W}) ,\end{aligned}$$

as $E[\mathbf{y}'(\mathbf{W}'\mathbf{A}+\mathbf{A}'\mathbf{W})\mathbf{y}] = \mathrm{tr}[(\mathbf{W}'\mathbf{A}+\mathbf{A}'\mathbf{W})E(\mathbf{y}\mathbf{y}')] = \mathrm{tr}[(\mathbf{W}'\mathbf{A}+\mathbf{A}'\mathbf{W})\sigma^2\mathbf{A}^{-1}(\mathbf{A}^{-1})']$
$= \sigma^2\mathrm{tr}[\mathbf{W}'\mathbf{A}\mathbf{A}^{-1}(\mathbf{A}^{-1})' + \mathbf{A}'\mathbf{W}\mathbf{A}^{-1}(\mathbf{A}^{-1})'] = \sigma^2[\mathrm{tr}(\mathbf{W}'(\mathbf{A}^{-1})') + \mathrm{tr}(\mathbf{A}'\mathbf{W}\mathbf{A}^{-1}(\mathbf{A}^{-1})')]$
$= \sigma^2[\mathrm{tr}(\mathbf{W}'(\mathbf{A}^{-1})' + \mathrm{tr}(\mathbf{W}\mathbf{A}^{-1}] = 2\sigma^2\mathrm{tr}(\mathbf{A}^{-1}\mathbf{W})$. This follows from using the permutation rule $\mathrm{tr}(\mathbf{ABCD})=\mathrm{tr}(\mathbf{BCDA})=\ldots$, giving $\mathrm{tr}(\mathbf{A}'\mathbf{W}\mathbf{A}^{-1}(\mathbf{A}^{-1})') = \mathrm{tr}(\mathbf{W}\mathbf{A}^{-1}(\mathbf{A}^{-1})'\mathbf{A}') = \mathrm{tr}(\mathbf{W}\mathbf{A}^{-1})$, and the rule that $\mathrm{tr}(\mathbf{A}') = \mathrm{tr}(\mathbf{A})$.

The relevant element of $\mathbf{V}$ is the upper diagonal element, $\mathbf{V}_{11}$. Using the relation between $\mathbf{I}$ and $\mathbf{V}$ and the rule of partitioned inverse, this element becomes

$$\mathbf{V}_{11} = \mathbf{I}_{11}^{-1}(1+\mathbf{I}_{12}F_2\mathbf{I}_{12}\,\mathbf{I}_{11}^{-1}) = \mathbf{I}_{22}/(\mathbf{I}_{11}\,\mathbf{I}_{22}-\mathbf{I}_{12}^2)$$

where $\mathbf{F}_2 = (\mathbf{I}_{22} - \mathbf{I}_{12}^2\,\mathbf{I}_{11}^{-1})^{-1}$. Finally, the ratio of the estimated value of ρ^* to the square root of $\mathbf{V}_{11}$ provides the desired τ value to be compared to the asymptotic t distribution (i.e. the standard normal distribution). If τ falls below the lower α percentile, then the null of nonstationarity is rejected at the α level.